

# A combinatorics quiz (solution continued)

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## 3 The fastest solution

At first we remark, that if we do not allow “great-grandmother”, then we get the —index-shifted— Fibonacci numbers. We see, that the numbers of the examined sequence (1, 2, 4, 7, 13, 24, ...) satisfy a similar property:  $7 = 4 + 2 + 1$ ,  $13 = 7 + 4 + 2$ ,  $24 = 13 + 7 + 4$ . As a matter of fact, the function

$$f(z) := C^{\{1,2,3\}}(z) = \frac{1}{1 - z - z^2 - z^3}$$

is the generating function of the Tribonacci numbers (index-shifted).<sup>1</sup> Thus we just have to calculate the first 100 or 1000 members of the sequence in order to get the 2 solutions. The next Python script does this in less than 0.2 seconds.

```
t1, t2, t3 = 1, 2, 4
i = 0
while i < 97:
    t = t1 + t2 + t3
    t1, t2, t3 = t2, t3, t
    i += 1
print t

while i < 997:
    t = t1 + t2 + t3
    t1, t2, t3 = t2, t3, t
    i += 1
print t
```

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<sup>1</sup>See e.g. R C Johnson: Fibonacci numbers and matrices, 2008, <http://maths.dur.ac.uk/~dma0rcj/PED/fib.pdf>, p. 19.